Sample Question Paper - 32
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

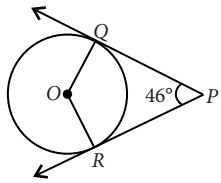
1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. If the numbers $2n - 1$, $3n + 2$ and $6n - 1$ are in A.P., then find n and hence find the numbers.

2. A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 40 m, then find the total area of the canvas required.

3. In the given figure, PQ and PR are two tangents to a circle with centre O. If $\angle QPR = 46°$, then what is the measure of $\angle QOR$?



OR

If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP.

4. The mean of a set of numbers is $\bar{x}$. If each number is multiplied by k, then find the mean of the new set.

5. Solve the following quadratic equation for x:
$9x^2 - 6b^2x - (a^4 - b^4) = 0$

6. For the following grouped frequency distribution, find the mode.

Class interval	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	4	10	11	21	12	9

OR

If the median of the series exceeds the mean by 3, then by what number the mode exceeds its mean?

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SECTION - B

7. If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find the value of k.

8. The ninth term of an A.P. is –32 and the sum of its eleventh and thirteenth terms is –94. Find the common difference of the A.P.

OR

The first and the last terms of an A.P. are 8 and 65 respectively. If the sum of all its terms is 730, then find its common difference.

9. Draw two tangents at the end points of the diameter of a circle of radius 3.5 cm. Are these tangents parallel?

10. A pole casts a shadow of length $2\sqrt{3}$ m on the ground, when the sun's elevation is 60°. Find the height of the pole.

SECTION - C

11. From the top of a 7 m high building, the angle of elevation of the top of a tower is 60°and the angle of depression of the foot of the tower is 30°. Find the height of the tower.

OR

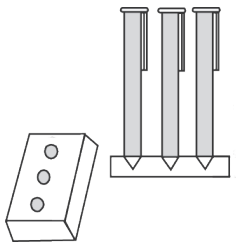
The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground is 45°. If the angle of elevation of the top of a flagstaff fixed at the top of the tower, at A is 60°, then find the height of the flagstaff. [Use $\sqrt{3} = 1.73$]

12. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

Case Study - 1

13. A carpenter used to make and sell different kinds of wooden pen stands like rectangular, cuboidal, cylindrical, conical. Aarav went to his shop and asked him to make a pen stand as explained below.

Pen stand must be of the cuboidal shape with three conical depressions, which can hold 3 pens. The dimensions of the cuboidal part must be 20 cm × 15 cm × 5 cm and the radius and depth of each conical depression must be 0.6 cm and 2.1 cm respectively.



Based on the above information, answer the following questions.

(i) Find the volume of the cuboidal part.

(ii) Find total volume of the conical depressions.

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Case Study - 2

14. A bread manufacturer wants to know the lifetime of the product. For this, he tested the life time of 400 packets of bread. The following tables gives the distribution of the life time of 400 packets.

Lifetime (in hours)	150-200	200-250	250-300	300-350	350-400	400-450	450-500
Number of packets (Cumulative frequency)	14	70	130	216	290	352	400

Based on the above information, answer the following questions.

(i) Find the average lifetime of a packet.

(ii) Find the median lifetime of a packet.

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Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

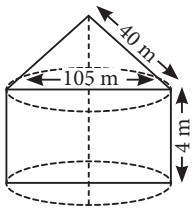
1. We have $2n - 1$, $3n + 2$ and $6n - 1$ are in A.P.

$\therefore \quad (3n + 2) - (2n - 1) = (6n - 1) - (3n + 2)$

$\Rightarrow \quad n + 3 = 3n - 3 \Rightarrow 6 = 2n \Rightarrow n = 3$

$\therefore$ Numbers are $2(3) - 1 = 5$, $3(3) + 2 = 11$, $6(3) - 1 = 17$.

2. Total area of canvas required = Curved surface area of cone + Curved surface area of cylinder



$= \pi rl + 2\pi rh$

$= \pi\left(\frac{105}{2}\right)40 + 2\pi\left(\frac{105}{2}\right)4$

$= \pi(2100 + 420) = \frac{22}{7} \times 2520 = 7920 \text{ m}^2$

3. Given, $\angle QPR = 46°$

We have, $OQ \perp PQ$ and $OR \perp RP$

[$\because$ Radius is $\perp$ to the tangent through the point of contact]

$\Rightarrow \quad \angle OQP = \angle ORP = 90°$

In quadrilateral $PQOR$, we have

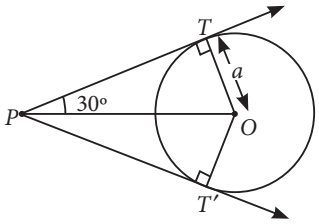
$\angle OQP + \angle QPR + \angle PRO + \angle ROQ = 360°$

$\Rightarrow \quad 90° + 46° + 90° + \angle ROQ = 360°$

$\Rightarrow \quad \angle ROQ = 360° - 226° = 134°$

OR

Since, tangents drawn from an external point are equally inclined to the line joining centre to that point.



$\therefore \quad \angle TPT' = 60° \Rightarrow \angle TPO = 30°$

Also, $OT \perp TP$

Now, in ΔTPO, $\sin 30° = \frac{OT}{OP}$

$\Rightarrow \quad \frac{1}{2} = \frac{a}{OP} \Rightarrow OP = 2a$

4. Let the numbers are $x_1, x_2, ..., x_n$.

$\therefore \quad \text{Mean} = \frac{x_1 + x_2 + ... + x_n}{n} = \bar{x}$ ...(i)

When given numbers are multiplied by k, then new observations are $kx_1, kx_2, ..., kx_n$.

$$\text{New mean} = \frac{kx_1 + kx_2 + ...kx_n}{n}$$

$$= \frac{k(x_1 + x_2 + ... + x_n)}{n} = k\bar{x}$$ (From (i))

5. We have, $9x^2 - 6b^2x - (a^4 - b^4) = 0$

$\Rightarrow \quad 9x^2 - 6b^2x - a^4 + b^4 = 0$

$\Rightarrow \quad \{(3x)^2 - 2(3x)b^2 + (b^2)^2\} - (a^2)^2 = 0$

$\Rightarrow \quad (3x - b^2)^2 - (a^2)^2 = 0$

$\Rightarrow \quad (3x - b^2 + a^2)(3x - b^2 - a^2) = 0$

$\Rightarrow \quad 3x - b^2 + a^2 = 0$ or $3x - b^2 - a^2 = 0$

$\Rightarrow \quad 3x = b^2 - a^2$ or $3x = b^2 + a^2$

$\Rightarrow \quad x = \frac{b^2 - a^2}{3}$ or $x = \frac{a^2 + b^2}{3}$

6. We observe that the class 15-18 has maximum frequency. Therefore, this is the modal class.

$l = 15, h = 3, f_0 = 11, f_1 = 21$ and $f_2 = 12$

$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 15 + \left(\frac{21 - 11}{42 - 11 - 12}\right) \times 3$

$\Rightarrow \quad \text{Mode} = 15 + \frac{10}{19} \times 3 = 15 + \frac{30}{19} = 16.58$

OR

Given, Median – Mean = 3 ...(i)

We know, Mode = 3 Median – 2 Mean

$\Rightarrow$ Mode – Mean = 3 Median – 3 Mean

= 3(Median – Mean)

$\Rightarrow$ Mode – Mean = 3 × 3 [Using (i)]

$\Rightarrow$ Mode – Mean = 9

Hence, the mode exceeds the mean by 9.

7. Since 2 is a root of $3x^2 + px - 8 = 0$

$\therefore \quad 3(2)^2 + p(2) - 8 = 0 \Rightarrow 2p + 4 = 0 \Rightarrow p = -2$

Putting $p = -2$ in $4x^2 - 2px + k = 0$, we get

$4x^2 + 4x + k = 0$ ...(i)

Since (i) has equal roots. $\therefore \quad D = 0$

$\Rightarrow \quad 4^2 - 4(4)(k) = 0 \Rightarrow 16 = 16k \Rightarrow k = 1$

8. Let the first term be a and d be the common difference of the A.P.

Given, $a_9 = -32 \Rightarrow a + 8d = -32$ ...(i)

Also, $a_{11} + a_{13} = -94$

$\Rightarrow \quad a + 10d + a + 12d = -94 \Rightarrow 2a + 22d = -94$

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$\Rightarrow\ a + 11d = -47$ ...(ii)

Subtracting (ii) from (i), we get

$-3d = 15 \ \Rightarrow\ d = -5$

OR

Let a and d denotes the first term and common difference respectively of the A.P.

Given, $a = 8$ and $l = 65 = a + (n-1)d$

$\Rightarrow\ 65 = 8 + (n-1)d \ \Rightarrow\ 57 = (n-1)d$ ...(i)

$$S_n = 730 \ \Rightarrow\ \frac{n}{2}(a+l) = 730$$

$$\Rightarrow\ n[8+65] = 1460 \ \Rightarrow\ n = \frac{1460}{73} = 20$$

Putting value of n in (i), we get $57 = (20-1)d$

$\Rightarrow\ 57 = 19d \ \Rightarrow\ d = 3$

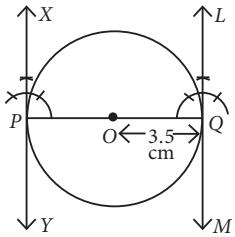
9. Steps of construction :

Step-I : Draw a circle with centre O and radius 3.5 cm.

Step-II : Draw the diameter POQ.

Step-III : Construct perpendiculars on PQ at both end points P and Q.

Thus, XPY and LQM are the two tangents at P and Q to the circle with centre O.



Since $\angle XPQ + \angle PQL$

$= 90° + 90° = 180°$

$\therefore\ XPY \parallel LQM$

10. Let AB be the pole of height h m and its shadow be BC.

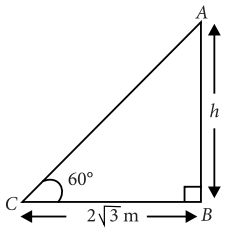
$BC = 2\sqrt{3}$ m, $\angle ACB = 60°$

In ΔABC,

$$\frac{h}{BC} = \tan 60° \ \Rightarrow\ \frac{h}{2\sqrt{3}} = \sqrt{3}$$

$\Rightarrow\ h = 2 \times 3 = 6$

$\therefore$ Height of the pole is 6 m.



11. Let $AB = 7$ m be the height of building and CD be the height of tower. Now, $AB = DE = 7$ m

Also, $BD = AE$

In ΔABD,

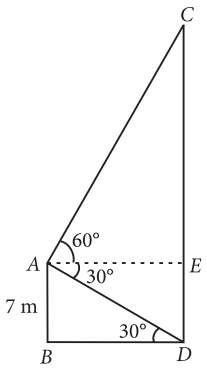
$$\frac{AB}{BD} = \tan 30° \ \Rightarrow\ \frac{7}{BD} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow\ BD = 7\sqrt{3} \text{ m} = AE$$

In ΔACE, $\frac{CE}{AE} = \tan 60°$

$$\Rightarrow\ \frac{CE}{7\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow\ CE = \sqrt{3} \times 7\sqrt{3} \text{ m} = 21 \text{ m}$$



$\therefore\ CD = CE + ED = (21 + 7)$ m $= 28$ m

Thus, the height of the tower is 28 m.

OR

Let $BD = h$ be the total height of tower and flagstaff

$\therefore$ In ΔDBA,

$$\tan 60° = \frac{h}{120}$$

$$\Rightarrow\ h = 120\sqrt{3} \text{ m}$$

and in ΔCBA,

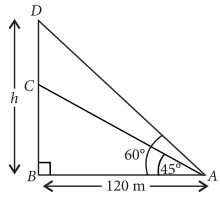
$$\tan 45° = \frac{CB}{120}$$

$\Rightarrow\ CB = 120$ m $\quad [\because \tan 45° = 1]$

$\therefore$ Height of flagstaff, $DC = DB - BC$

$$= 120\sqrt{3} - 120 = 120(\sqrt{3} - 1)$$

$= 120(1.73 - 1) = 120(0.73) = 87.6$ m

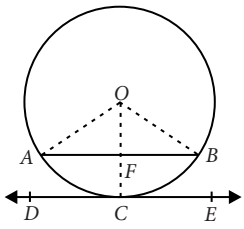


12. Given : A circle with centre O and C is the mid point of arc ACB and DE is a tangent to the circle.

To prove : $AB \parallel DE$

Construction : Join OA, OB and OC.

Proof : Since C is the midpoint of arc ACB.



$\therefore\ \angle AOF = \angle BOF$

($\because$ OA and OB are equally inclined with OC)

Now, in ΔOAF and ΔOBF,

$OA = OB$ (Radii of the circle)

$\angle AOF = \angle BOF$ (Proved above)

$OF = OF$ (Common)

$\therefore\ \Delta OAF \cong \Delta OBF$ (By SAS criterion)

$\Rightarrow\ \angle AFO = \angle BFO$ (By CPCT)

Now, $\angle AFO + \angle BFO = 180°$ (Linear pair)

$\Rightarrow\ 2\angle AFO = 180° \ \Rightarrow\ \angle AFO = 90°$

Also, $\angle OCD = 90°$

Tangent is perpendicular to radius through the point of contact.

$\therefore\ \angle AFO = \angle OCD$ (Each 90°)

But these are corresponding angles $\therefore\ AB \parallel DE$

13. (i) Volume of cuboidal part $= l \times b \times h$

$= (20 \times 15 \times 5)$ cm^3 $= 1500$ cm^3

(ii) Radius of conical depression, $r = 0.6$ cm

Height of conical depression, $h = 2.1$ cm

$\therefore$ Total volume of conical depressions $= 3 \times \frac{1}{3}\pi r^2 h$

$$= \frac{22}{7} \times 0.6 \times 0.6 \times 2.1 = \frac{2376}{1000} = 2.376 \text{ cm}^3$$

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14. (i)

Lifetime (in hours)	Class mark (x_i)	f_i	$d_i = x_i - A$	$f_i d_i$
150-200	175	14	−150	−2100
200-250	225	56	−100	−5600
250-300	275	60	−50	−3000
300-350	325 = A	86	0	0
350-400	375	74	50	3700
400-450	425	62	100	6200
450-500	475	48	150	7200
Total		400		6400

∴ Average lifetime of a packet

$$= A + \frac{\sum f_i d_i}{\sum f_i} = 325 + \frac{6400}{400} = 341 \text{ hrs}$$

(ii) Here, $N = 400 \Rightarrow \frac{N}{2} = 200$

Also, cumulative frequency for the given distribution are 14, 70, 130, 216, 290, 352, 400

∴ *c.f.* just greater than 200 is 216, which is corresponding to the interval 300-350.

$l = 300$, $f = 86$, *c.f.* = 130, $h = 50$

$$\therefore \text{ Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h = 300 + \left(\frac{200 - 130}{86}\right) \times 50$$

= 300 + 40.697 = 340.697 ≈ 340 hrs (approx.)

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